Kumon Mathematics Programme up to J-level

7A-5A

Counting
Number Recognition
Line Tracing
Number Writing

4A-A B

Number Writing Exercises

1 2 3 4 5
6 7 8 9 10

Addition

$2 + 1 = 3$

$24 + 3 = 27$

$8 + 5 = 13$

$\begin{array}{r} 58 \\ +\ 95 \\ \hline 153 \end{array}$

Subtraction

$13 - 5 = 8$

$\begin{array}{r} 113 \\ -\ 105 \\ \hline 8 \end{array}$

C D

Multiplication

$\begin{array}{r} 39 \\ \times\ 6 \\ \hline 234 \end{array}$

$\begin{array}{r} 39 \\ \times\ 66 \\ \hline 234 \\ 234 \\ \hline 2574 \end{array}$

$\begin{array}{r} 65 \\ \times\ 39 \\ \hline 585 \\ 195 \\ \hline 2535 \end{array}$

Division

$15\overline{)113}\ 7\text{R}8$

$\begin{array}{r} 105 \\ \hline 8 \end{array}$

$21 \div 3 = 7$

Fractions

$6\overline{)97}\ 16\text{R}1$

$\frac{97}{6} = 16\frac{1}{6}$

$16\frac{1}{6} = \frac{97}{6}$

$\frac{7}{21} = \frac{1}{3}$

E F

Addition and Subtraction of Fractions

- $\frac{3}{4} + \frac{1}{6} = \frac{9}{12} + \frac{2}{12} = \frac{11}{12}$
- $\frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$
- $\frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{4}{12} + \frac{3}{12} + \frac{2}{12} = \frac{9}{12} = \frac{3}{4}$

Least Common Multiple of 3, 4 and 6 is 12.

- $2\frac{1}{4} \times \frac{2}{3} = \frac{\cancel{9}^{3}}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{\cancel{3}_{1}} = 1\frac{1}{2}$
- $\frac{8}{21} \div \frac{4}{7} = \frac{\cancel{8}^{2}}{\cancel{21}_{3}} \times \frac{\cancel{7}^{1}}{\cancel{4}_{1}} = \frac{2}{3}$

Four Operations

$\frac{5}{6} \times \frac{1}{3} - \frac{7}{12} \times \frac{1}{3} = \frac{5}{18} - \frac{7}{36} = \frac{10}{36} - \frac{7}{36} = \frac{3}{36} = \frac{1}{12}$

$(\frac{5}{6} - \frac{7}{12}) \times \frac{1}{3} = (\frac{10}{12} - \frac{7}{12}) \times \frac{1}{3} = \frac{3}{12} \times \frac{1}{3} = \frac{1}{12}$

Solving Word Problems

Cindy leaves her house by bike at 1 p.m., and Rosy leaves her house by bike at 2 p.m. Their houses are 120 km apart from each other. Cindy travels 15 km in 1 hour, and Rosy travels 20 km in 1 hour. What time will they meet?

(Solution)

By the time Rosy leaves, Cindy has already travelled 15 km; therefore, the distance between the two is 120 – 15 = 105 km.
Cindy and Rosy then travel 35 km in 1 hour; therefore, it will take 105 ÷ 35 = 3 hours for them to meet; the time will be 5 p.m.

G

Four Operations with Positive and Negative Numbers

- $-\frac{3}{4} - \frac{1}{6} = -\frac{9}{12} - \frac{2}{12} = -\frac{11}{12}$
- $2\frac{1}{4} \times (-\frac{2}{3}) = -\frac{\cancel{9}^{3}}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{\cancel{3}_{1}} = -1\frac{1}{2}$

Values of Algebraic Expressions

If... $x = -\frac{1}{4}$, $y = \frac{1}{6}$

Find the value of the following expressions

$3x - y = 3 \times (-\frac{1}{4}) - \frac{1}{6}$

$= -\frac{3}{4} - \frac{1}{6} = -\frac{11}{12}$

Simplifying Algebraic Expressions

- $\frac{1}{3}a - \frac{1}{4}a = \frac{4}{12}a - \frac{3}{12}a = \frac{a}{12}$

Equations

- $\begin{array}{rcl} 5x - 13 & = & -8x + 26 \\ 5x + 8x & = & 26 + 13 \\ 13x & = & 39 \\ x & = & \frac{39}{13} = 3 \end{array}$
- $\frac{3}{4}x - 2 = \frac{x}{6} + \frac{1}{3}$

Multiply both sides by 12, i.e. the lowest common denominator of 3, 4 and 6

$\begin{array}{rcl} 9x - 24 & = & 2x + 4 \\ 7x & = & 28 \\ x & = & 4 \end{array}$

Application of Equations

Kate has 18 pencils and Lucy has 12 pencils. If Kate gives Lucy x pencils, they will both have the same number of pencils. How many is x?

After Kate gives Lucy x pencils, Kate has (18-x) pencils and Lucy has (12+x) pencils. Because they have the same number of pencils,

$\begin{array}{rcl} 18 - x & = & 12 + x \\ 6 & = & 2x \\ x & = & 3 \end{array}$

H

Simultaneous Linear Equations in 2 Variables

$\begin{cases} 5x + 7y = 3 \cdots\cdots ① \\ 3x + 14y = 6 \cdots\cdots ② \end{cases}$

The first step is to eliminate one of the variables using the lowest common denominator.

$\begin{array}{rl} ① \times 2 : & 10x + 14y = 6 \cdots\cdots ③ \\ & 3x + 14y = 6 \cdots\cdots ② \end{array}$

$③ - ② : \quad 7x = 0$

$x = 0 \cdots\cdots ④$

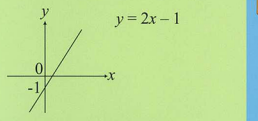
Substituting ④ into ①, $y = \frac{3}{7}$

$(x, y) = (0, \frac{3}{7})$

Inequalities

- $\begin{array}{rcl} 5x - 13 & > & -8x + 26 \\ 5x + 8x & > & 26 + 13 \\ 13x & > & 39 \\ x & > & 3 \end{array}$

Functions and Graphs



$y = 2x - 1$

Polynomials

- $-4xy(3x^2 - xy + 2y^2)$
- $= -12x^3y + 4x^2y^2 - 8xy^3$

I

Multiplication of Polynomials

- $(x + 5)(x + 8)$
  $= x^2 + 5x + 8x + 40$
  $= x^2 + 13x + 40$

Factorisation

- $6a(x - y) + 9b(x - y)$
  $= 3(x - y)(2a + 3b)$
- $x^2 + 13x + 40$
  $= (x + 5)(x + 8)$

Square Roots

- $\sqrt{18} + \sqrt{50} = 3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$
- $\frac{1}{\sqrt{3} + 1} = \frac{\sqrt{3} - 1}{(\sqrt{3} + 1)(\sqrt{3} - 1)} = \frac{\sqrt{3} - 1}{2}$

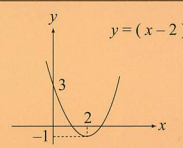
Quadratic Equations

- $x^2 + 13x + 40 = 0$
  $(x + 5)(x + 8) = 0$
  $x = -5, -8$
- $3x^2 + 9x - 2 = 0$

$x = \frac{-9 \pm \sqrt{9^2 - 4 \times 3 \times (-2)}}{2 \times 3}$

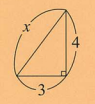
$= \frac{-9 \pm \sqrt{105}}{6}$

Graphs of Quadratic Functions



$y = (x - 2)^2 - 1$

The Pythagorean Theorem



$x^2 = 4^2 + 3^2$
$= 16 + 9$
$= 25$
$x = 5$

J

Expansion of Polynomial Products

- $(a + b + c)^2$
  $= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Factorisation

- $8x^6 + 19x^3y^3 - 27y^6$
  $= (x^3 - y^3)(8x^3 + 27y^3)$
  $= (x - y)(x^2 + xy + y^2)$
  $(2x + 3y)(4x^2 - 6xy + 9y^2)$

Fractional Expressions

- $\frac{x + 1}{x^2 - x - 2} = \frac{x + 1}{(x - 2)(x + 1)} = \frac{1}{x - 2}$

Irrational Numbers

- $\sqrt{5 + 2\sqrt{6}} = \sqrt{3} + \sqrt{2}$

Quadratic Equations and Complex Numbers

- $2x^2 + 5x + 4 = 0$

$x = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times 4}}{2 \times 2}$

$= \frac{-5 \pm \sqrt{-7}}{4} = \frac{-5 \pm \sqrt{7}i}{4}$

The Discriminant and Root Coefficient Relationships

- $3x^2 - 8x + k = 0$

Determine the value of k so that the following equation has a repeated solution.

$D' = 4^2 - 3k$
$16 - 3k = 0$
$k = \frac{16}{3}$

Remainder Theorem

- $f(x) = x^3 + 4x^2 - 5x + 3$

Divide by $x - 2$

$f(2) = 8 + 16 - 10 + 3 = 17$

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